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Crossing a Coupling Spin Resonance With an RF Dipole[1]

M. Bai, T. Roser

Brookhaven National Laboratory, Upton, NY 11973, U.S.A

Abstract. In accelerators, due to quadrupole roll errors and solenoid fields, the polarized proton acceleration often encounters coupling spin resonances. In the Brookhaven AGS, the coupling effect comes from the solenoid partial snake which is used to overcome imperfection resonances. The coupling spin resonance strength is proportional to the amount of coupling as well as the strength of the corresponding intrinsic spin resonance. The coupling resonance can cause substantial beam polarization loss if its corresponding intrinsic spin resonance is very strong. A new method of using an horizontal RF dipole to induce a full spin flip crossing both the intrinsic and its coupling spin resonances is studied in the Brookhaven's AGS. Numerical simulations show that a full spin flip can be induced after crossing the two resonances by using a horizontal RF dipole to induce a large vertical coherent oscillation.

I INTRODUCTION

In an accelerator, particles undergo betatron oscillations in both horizontal and vertical planes while they circulate around the machine. In a perfect machine, both oscillations are independent of each other. However, this independence can be broken if there is any quadrupole roll errors or solenoid fields. In this case, the horizontal motion is coupled to the vertical oscillation. Unlike the uncoupled case, the frequency spectrum of the betatron oscillation in either of the two transverse plane then consists of two components ν_1 and ν_2 given by [1,2]

$$\nu_1 = \frac{1}{2}(\nu_x + \nu_z) + \frac{1}{2}\sqrt{(\nu_x - \nu_z)^2 + \Delta Q^2_{min}} \rightarrow \nu_x;\ \text{without coupling} \tag{1}$$

$$\nu_2 = \frac{1}{2}(\nu_x + \nu_z) - \frac{1}{2}\sqrt{(\nu_x - \nu_z)^2 + \Delta Q^2_{min}} \rightarrow \nu_z;\ \text{with couple} \tag{2}$$

where ν_x and ν_z are the unperturbed horizontal and vertical tunes. ΔQ_{min} is the minimum tune split between the two eigen tunes when $\nu_x = \nu_z$ and is proportional to the coupling strength [3]. With weak coupling,

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$$\nu_1 \simeq \nu_x \tag{3}$$

$$\nu_2 \simeq \nu_z. \tag{4}$$

In the Brookhaven AGS, the main coupling source comes from the solenoid partial snake which is used to overcome the imperfection spin resonances in the AGS [4]. The minimum tune split ΔQ_{min} from the 5% partial snake is about 0.015.

In a coupled machine, in addition to the intrinsic spin resonance at $G\gamma = kP \pm \nu_2$ ($G\gamma = kP \pm \nu_z$ without coupling) [5], the vertical betatron oscillation also drives a coupling spin resonances at $G\gamma = kP \pm \nu_1$ [3]. The strength of the coupling resonance ϵ_{ν_x} is proportional to the amount of the coupling and it is given by

$$\epsilon_{\nu_x} \propto C_x \sqrt{\varepsilon_u} \epsilon_{\nu_z} \tag{5}$$

where ϵ_{ν_z} is the strength of the adjacent intrinsic spin resonance and C_x is the coupling coefficient. For a fully coupled machine, $\nu_x = \nu_z$ and $C_x = 1$. For a decoupled machine, $C_x = 0$. ε_u is the beam emittance in the eigen direction [6] and equals the horizontal beam emittance if $C_x = 0$.

In the AGS, there are four strong intrinsic spin resonances at $0 + \nu_z$, $12 + \nu_z$ and $36 \pm \nu_z$ [5]. Traditionally in the AGS, the beam polarization loss at the coupling resonances is minimized by separating the horizontal and vertical tunes. The coupling resonances around these four strong intrinsic resonances can produce about 35% polarization losses with the normal AGS polarized proton setting [7,8]. In order to achieve 70% polarization in the AGS, one needs to minimize the polarization loss at the coupling resonances. Since they are adjacent to the intrinsic resonances, it is very difficult to use the vertical RF dipole [9] to obtain full spin flips at both the intrinsic and the coupling resonances.

Analogous to the method of using a vertical RF dipole at the intrinsic spin resonance, one should also expect to obtain a full spin flip by inducing a strong artificial resonance if the intrinsic and its coupling spin resonances are fully overlapped. Because of the coupling effect, the two spin resonances can never be brought closer than the minimum tune split ΔQ_{min}. However, ΔQ_{min} in general is small and a full spin flip still should be achievable if the induced resonance is strong enough. In a fully coupled machine, the unperturbed tunes are equal and the intrinsic and the coupling resonances are equally strong and located on either side of the unperturbed betatron tune at a distance of half of ΔQ_{min}.

Unlike using a vertical RF dipole to obtain a vertical coherence in an uncoupled machine [10], the vertical coherence is excited by a horizontal RF dipole instead in a fully coupled machine. This can be understood by solving the differential equation of a coupled driven oscillator

$$\begin{aligned} x'' + (2\pi f \nu)^2 x + qz &= Acos(2\pi f \nu_m \theta) \\ z'' + (2\pi f \nu)^2 z + qx &= 0. \end{aligned} \tag{6}$$

Here f is the revolution frequency, ν_m is the modulation tun, q is the coupling strength and A is the amplitude of the driving term. When the modulation tune

ν_m equals the unperturbed tune ν, the solution of Eq. 6 gives $x = 0$ and a pure vertical oscillation $z = \frac{A}{q} cos 2\pi f \nu_m \theta$.

Fig. 1 shows numerical spin tracking results at $G\gamma = 36 + \nu_z$. The dotted line shows the result with the nominal AGS tune setting ($\nu_x = 8.8, \nu_z = 8.7$) and no correction scheme for the intrinsic spin resonance. In this case, the depolarization at the coupling resonance is obvious. The solid line is the result of using a horizontal RF dipole with the horizontal and vertical betatron tunes set at 8.7. Due to the coupling from the solenoid partial snake, the two betatron tunes are split by 0.0144. The horizontal RF dipole tune was set to 0.3. With a horizontal RF dipole amplitude 28.0 G-m, a full spin flip was achieved.

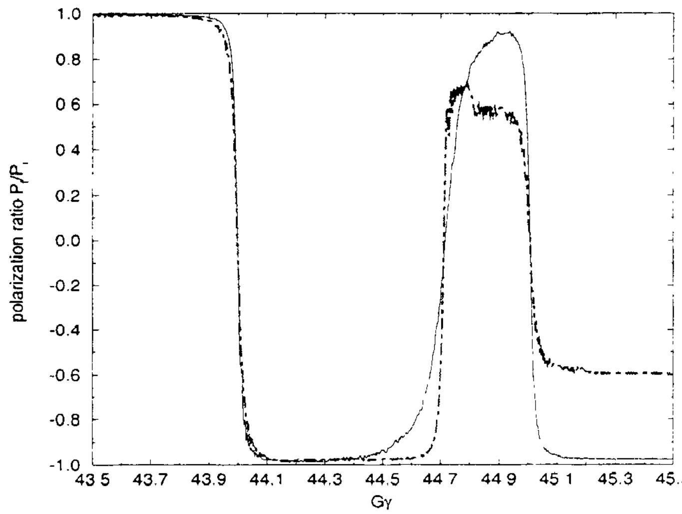

FIGURE 1. The two curves are the calculated polarization ratio P_f/P_i as a function of energy. The solid line is for the case of a fully coupled machine and a horizontal RF dipole was used to obtain an adiabatic vertical coherence. The dotted line is the result of a weakly coupled machine with the two betatron tunes set 0.1 apart. No correction scheme was used at the $G\gamma = 36 + \nu_z$ intrinsic spin resonance. For both cases, the horizontal and vertical emittance are 20π mm-mrad and 10π mm-mrad respectively.

II EXPERIMENTAL RESULTS

The method of using a horizontal RF dipole to excite a vertical coherence to cross the coupling spin resonance was tested in the AGS during the 2000 RHIC polarized proton commissioning run. The polarized H^- beam was pre-accelerated in the 200 MeV LINAC and then stripped and injected into the Booster. It was then injected into the AGS at $G\gamma = 4.7$ and then accelerated up to $G\gamma = 46.5$. In the AGS, the nominal tune setting is $\nu_x = 8.8$ and $\nu_z = 8.7$.

During the experiment, the AGS skew quadrupoles were all set to 17 A. Due to a hardware limit, the partial snake strength at $G\gamma = 36 + \nu_z$ is actually only about 3.5% instead of 5%. The combined effect of the skew quadrupoles and the weaker snake gave a smaller minimum tune spilt ΔQ_{min} of 0.007. The horizontal RF dipole was set in the middle of the two betatron tunes ν_1 and ν_2. The turn by turn beam position monitor data confirmed that a vertical coherence was excited without horizontal response as shown in the two left plots of Fig. 2. The horizontal response was not zero once the RF dipole tune deviated from the average of the two eigen tunes as shown on the right of Fig. 2.

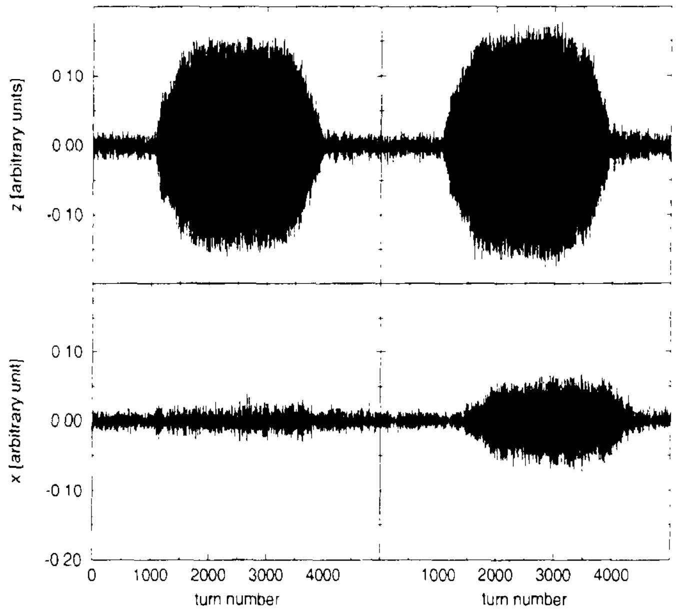

FIGURE 2. The top and bottom plots on the left are vertical and horizontal turn-by-turn beam position data when the horizontal RF dipole modulation tune $\nu_m = \frac{1}{2}(\nu_1 + \nu_2)$. As shown, no horizontal coherence was excited. The two plots on the right correspond to the case where the horizontal modulation tune $\nu_m \neq \frac{1}{2}(\nu_1 + \nu_2)$ and the horizontal coherence was no longer zero.

Table 1 shows the comparison of the measured beam asymmetries of using vertical RF dipole, no correction and using horizontal RF dipole at $G\gamma = 36 + \nu_z$. Comparing the measured asymmetry when using the horizontal RF dipole with the case of no correction, it is clear that the horizontal RF dipole did help to recover the beam polarization. However, the excited coherence was not optimized and about 70% beam emittance growth was observed. Because of limitations of the AGS sextupole power supplies, we could not achieve small chromaticities in both planes and obtain a fully adiabatic excitation. This is the most likely reason that the horizontal RF dipole did not recover 100% beam polarization as expected.

TABLE 1. measured asymmetry

	measured asymmetry (x10^{-3})	condition
1	1.50± 0.04	with vertical RF dipole
2	1.25± 0.1	with horizontal RF dipole
3	0.067± 0.063	no correction

III CONCLUSION

It has been demonstrated in the AGS that in a fully coupled machine, a vertical coherence can be excited by an horizontal RF dipole. Although beam polarization was improved, we think the residual polarization loss was due to the not fully adiabatic beam motion.

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